Acceleration, Gamma, and Theta Guidance for Abort Landing in a Windshear

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This paper is concerned with the guidance of abort landing trajectories in a windshear. First, optimal trajectories are determined by minimizing the peak value of the altitude drop. Then, two guidance schemes, approximating the optimal trajectories, are developed: acceleration guidance (based on the relative acceleration) and gamma guidance (based on the absolute path inclination). From numerical experiments, it appears that both the acceleration guidance and the gamma guidance yield trajectories that are close to the optimal trajectory. In addition, a theta guidance scheme (modified constant pitch guidance) is developed that is superior to the constant pitch guidance in terms of the altitude loss and the survival capability in severe windshears.

Nomenclature

g	= acceleration of gravity, ft/s^2
h	= altitude, ft
K	= gain coefficient, dimensionless or dimensional, depending on the feedback control law
V	= relative velocity, fps
W	= weight, lb
W_h	= h-component of wind velocity, fps
W_x	= x-component of wind velocity, fps
x	= horizontal distance, ft
α	= relative angle of attack (wing), rad
β	= engine power setting
γ	= relative path inclination, rad
γ_e	= absolute path inclination, rad
θ	= pitch attitude angle (wing), rad
λ	= wind intensity parameter
τ	= final time, s

I. Introduction

WHEN the pilot of an aircraft on an approach path detects an inadvertent encounter with a windshear, two possibilities arise: 1) penetration landing and 2) abort landing. The first option was considered in Ref. 1, appearing in this issue. The second option is considered here (for more details, see Ref. 2).

Generally speaking, abort landing is a safer procedure than penetration landing, except in the extreme case where the windshear is detected very close to the ground. For abort landing, we note that a constant pitch guidance scheme was introduced and discussed in Refs. 3–5.

For abort landing, optimal trajectories (trajectories minimizing the peak value of the altitude drop) were studied in Ref. 6. Based on the properties of the optimal trajectories, two guidance schemes were developed in Ref. 7: 1) Target altitude guidance, in which the target altitude is slightly above the minimum altitude of the optimal trajectory; and 2) safe target altitude guidance, in which the target altitude is slightly above the minimum altitude of the optimal trajectory for $\Delta W_x = 140$ fps. This is the maximum wind velocity difference ever recorded, if one excludes the 1983 windshear episode at Andrews Air Force Base. A drawback of scheme 1 is that it requires the a priori knowledge of a global quantity—the total wind velocity difference ΔW_x; a drawback of scheme 2 is that it makes a rather pessimistic assumption on the total wind velocity difference; as a consequence, although the safe target altitude guidance performs almost optimally for strong-to-severe windshears ($\Delta W_x = 120$–140 fps), it does not operate near optimality for weak-to-moderate windshears ($\Delta W_x = 80$–100 fps).

To correct the preceding drawbacks, we develop in this paper an acceleration guidance scheme and a gamma guidance scheme akin to those introduced in Refs. 8 and 9 for the takeoff problem. The intent is to appoximate the optimal trajectory while using only local measurements on the wind acceleration, the downdraft, and the state of the aircraft. In addition, starting from the constant pitch guidance of Refs. 3–5, we develop a theta guidance (modified constant pitch guidance) that is superior to the constant pitch guidance in two aspects: altitude drop and survival capability in strong-to-severe windshears.

Since this paper is a companion to Ref. 1, the common parts are omitted for the sake of brevity, in particular, the equations of motion [Eqs. (1–4) of Ref. 1], the description of the thrust and the aerodynamic forces [Eqs. (11) and (12) of Ref. 1], and the description of the wind model [Eqs. (13) and (14) and Fig. 1 of Ref. 1]. An analogous remark refers to the angle-of-attack bounds [inequalities (6) and Eqs. (7) of Ref. 1] and the power setting bounds [inequalities (8) of Ref. 1].

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II. System Description

The numerical examples of this paper refer to a Boeing B-727 aircraft powered by three JT8D-17 turbofan engines. It is assumed that the runway is located at sea-level altitude, the ambient temperature is 100°F, the gear is down, the flap setting is $\delta_F = 30$ deg, and the landing weight is $W = 150{,}000$ lb.

The aircraft is assumed to be in quasisteady flight along the approach path. At the windshear onset, the power setting is increased to the maximum value at a constant time rate. Hence, the power setting distribution is represented by

$$\beta = \beta_0 + \dot{\beta}_0 t, \qquad 0 \le t \le \sigma \tag{1a}$$

$$\beta = 1, \qquad \sigma \le t \le \tau \tag{1b}$$

where $\sigma = (1 - \beta_0)/\dot{\beta}_0$. Here, β_0 is the initial power setting, $\dot{\beta}_0$ is the constant time rate of increase of the power setting (in the examples, $\dot{\beta}_0 = 0.2/\text{s}$), σ is the time at which the maximum power setting is reached, and τ is the final time.

The wind model of Ref. 1 is retained, and the following values of the wind intensity parameter λ are considered:

$$\lambda = 1.0, \quad 1.2, \quad 1.4 \tag{2a}$$

These values correspond to the following wind velocity differences:

$$\Delta W_x = 100\lambda = 100, \quad 120, \quad 140 \text{ fps} \tag{2b}$$

We recall that the wind velocity difference ΔW_x is defined to be the maximum tail wind minus the maximum head wind.

Initial Conditions

For the examples reported in this paper, the following initial conditions are assumed:

$$x_0 = 0 \text{ ft} \tag{3a}$$

$$h_0 = 200, \quad 600, \quad 1000 \text{ ft} \tag{3b}$$

$$V_0 = 239.7 \text{ fps} \tag{3c}$$

$$\gamma_{e0} = -3.0 \text{ deg} \tag{3d}$$

The initial velocity [Eq. (3c)] is Federal Aviation Administration certification velocity V_{ref} augmented by 10 knots. The initial absolute path inclination [Eq. (3d)] is the standard glide slope used in the approach maneuver. The initial values γ_0, α_0, θ_0, and β_0 are computed using Eqs. (3) and the assumption of quasisteady flight prior to the windshear onset.

Final Conditions

For the examples reported in this paper, the following final conditions are assumed:

$$\tau = 40 \text{ s} \tag{4a}$$

$$\gamma_\tau = 7.431 \text{ deg} \tag{4b}$$

The final time [Eq. (4a)] is about twice the duration of the windshear encounter, $\Delta t = 22$ s. Note that the final relative path inclination [Eq. (4b)] corresponds to the steepest climb condition in quasisteady flight. Precise satisfaction of Eq. (4b) is imposed on the optimal trajectories; approximate satisfaction of Eq. (4b) is required in the guidance schemes.

III. Optimal Trajectories

For the abort landing problem, optimal trajectories are generated by minimizing the peak value of the altitude drop, that is, by minimizing the following performance index:

$$I = \max_t (h_R - h), \qquad 0 \le t \le \tau \tag{5}$$

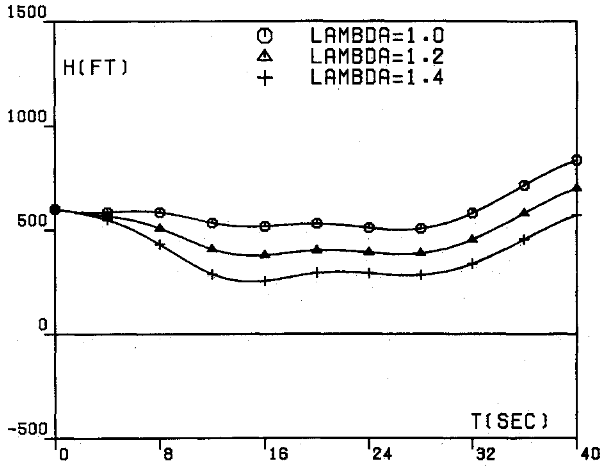

Fig. 1a Optimal trajectories, $h_0 = 600$ ft: altitude h vs time t.

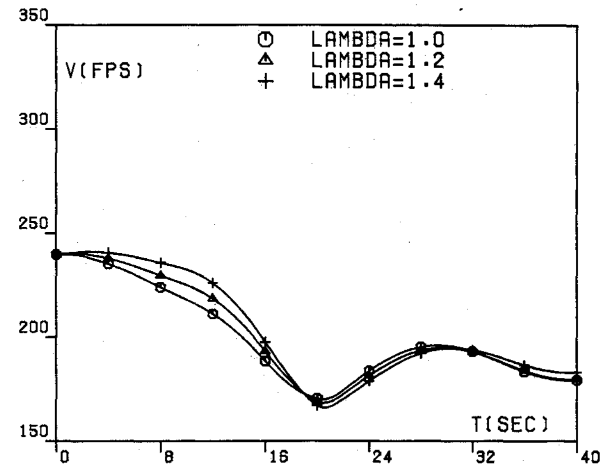

Fig. 1b Optimal trajectories, $h_0 = 600$ ft: relative velocity V vs time t.

where h_R is a constant reference altitude and h is the instantaneous altitude. By exploiting a well-known theorem of functional analysis, the minimization of the functional (5) is replaced by the minimization of the integral performance index

$$J = \int_0^\tau (h_R - h)^q \mathrm{d}t \tag{6}$$

for large values of the positive, even exponent q.

In the minimization process, satisfaction of the following constraints is enforced: the dynamical constraints (1-4) of Ref. 1; the angle-of-attack bounds (6) of Ref. 1, converted into Eqs. (7) of Ref. 1; the power setting distribution of this paper [Eqs. (1)]; and the boundary conditions of this paper [Eqs. (3) and (4)]. This means that a transition from descending flight to ascending flight is desired.

IV. Numerical Results on Optimal Trajectories

Using the data of Sec. II, optimal trajectories were computed. Several combinations of initial altitudes and windshear intensities were considered [see Eqs. (2) and (3)]. For each combination, the optimal control problem was solved with the sequential gradient-restoration algorithm, employed in conjunction with the dual formulation (DSGRA).[10,11] From the numerical solutions, certain general conclusions are apparent.

1) The optimal trajectory includes three branches in sequence: an initial branch, flown entirely in the shear region of the trajectory; a central branch, flown partly in the shear region and partly in the aftershear region; and a final branch, flown entirely in the aftershear region of the trajectory. For strong-to-severe windshears, the initial branch is descending, the central branch is nearly horizontal, and the final branch is ascending.

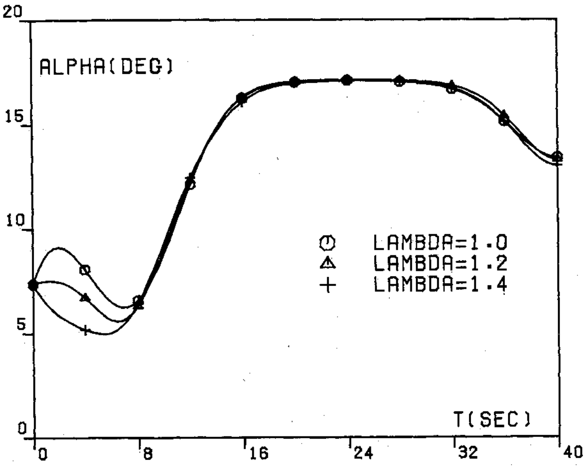

Fig. 1c Optimal trajectories, $h_0 = 600$ ft: relative angle of attack α vs time t.

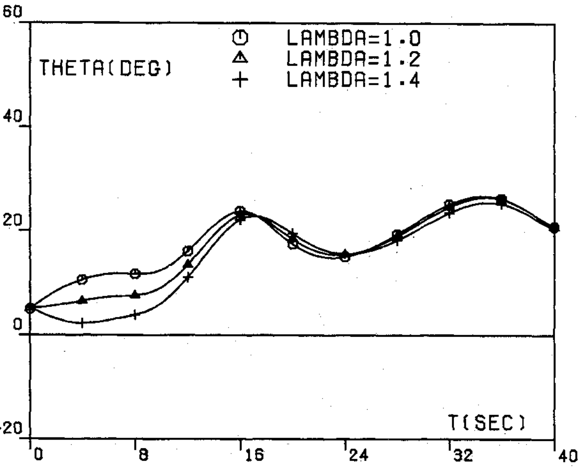

Fig. 1d Optimal trajectories, $h_0 = 600$ ft: pitch attitude angle θ vs time t.

2) The peak altitude drop depends on the initial altitude and the windshear intensity; it increases as the initial altitude increases and the windshear intensity increases.

3) The point of minimum velocity is achieved at the end of the shear. In the range of initial altitudes and windshear intensities given by Eqs. (2) and (3), the minimum velocity is nearly independent of h_0 and λ. Also, the minimum velocity is near to (but is not equal to, because of dynamical effects and windshear effects) the stick-shaker velocity in quasisteady level flight, $V_* = 179.3$ fps.

4) For strong-to-severe windshears, the angle of attack exhibits an initial decrease, followed by a gradual, sustained increase. The peak value of the angle of attack is achieved near the end of the shear.

5) For strong-to-severe windshears, the pitch attitude angle maintains relatively small values in the neighborhood of the initial point (it decreases initially for some combinations of h_0 and λ); then, it increases gradually in the nearly horizontal branch of the trajectory. Larger values of the pitch attitude angle are achieved toward the end of the shear.

For a particular case, namely, $h_0 = 600$ ft, the numerical results are presented in Fig. 1. This figure includes four parts: the altitude h (Fig. 1a); the relative velocity V (Fig. 1b); the relative angle of attack α (Fig. 1c); and the pitch attitude angle θ (Fig. 1d).

V. Acceleration Guidance

Here we present an acceleration guidance scheme, based on the relative acceleration, whose objective is to approximate the behavior of the optimal trajectory in a windshear. While the optimal trajectory is based on global information on the wind distribution, the acceleration guidance scheme relies on local information on the horizontal wind acceleration, the downdraft, and the state of the aircraft.

The acceleration guidance scheme is a modification of the guidance scheme bearing the same name, already developed for takeoff trajectories.[8] The modification accounts for these facts and ideas: 1) in abort landing, the initial path inclination is descending; in takeoff, it is ascending; 2) an abort landing trajectory can be viewed as a transition from a descending trajectory to an ascending trajectory; 3) if the abort landing maneuver is initiated at high altitude, containing the velocity loss should have priority over containing the altitude drop; and 4) if the abort landing maneuver is initiated at low altitude, containing the altitude drop should have priority over containing the velocity loss.

Guidance Law

The guidance law includes three parts, one pertaining to branch 1 (descent guidance), one pertaining to branch 2 (recovery guidance), and one pertaining to branch 3 (ascent guidance).

For the descending branch, the key property is that the velocity decrease is relatively low. Hence, the descent guidance law has the form

$$\dot{V}/g = 0 \tag{7}$$

For the nearly horizontal branch, the key property is that the instantaneous acceleration is approximately proportional to the shear/downdraft factor F, introduced in Ref. 8. Hence, the recovery guidance law has the form

$$\dot{V}/g + CF = 0 \tag{8a}$$

$$F = (\dot{W}_x/g)\cos\gamma + (\dot{W}_h/g)\sin\gamma - W_h/V \tag{8b}$$

where C is a constant, having the typical value $C = 0.5$. Note that, if γ is small and if $|\dot{W}_h\gamma/\dot{W}_x| << 1$, the shear/downdraft factor simplifies to

$$F \cong \dot{W}_x/g - W_h/V \tag{8c}$$

For the ascending branch, the key property is that the instantaneous path inclination tends to a positive value. This objective can be achieved indirectly with an ascent guidance law of the form

$$V - AV_0 = 0 \tag{9}$$

where V_0 is the initial velocity and $A \leq 1$ is a constant to be specified in advance (for instance, $A = 5/6$).

Feedback Control

The guidance laws [Eqs. (7-9)] can be implemented in the feedback control forms that follow.

For the descending branch, the descent guidance is implemented in the feedback control form

$$\alpha - \tilde{\alpha}(V) = K_1(\dot{V}/g - 0) \tag{10a}$$

$$\alpha \leq \alpha_*, \qquad -\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_* \tag{10b}$$

Here, $K_1 = 10$ rad is the gain coefficient and $\tilde{\alpha}(V)$ is the nominal angle of attack, whose conception is discussed in Ref. 8.

For the nearly horizontal branch, the recovery guidance is implemented in the feedback control form

$$\alpha - \tilde{\alpha}(V) = K_2(\dot{V}/g + CF) \tag{11a}$$

$$F \cong \dot{W}_x/g - W_h/V \tag{11b}$$

$$\alpha \leq \alpha_*, \qquad -\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_* \tag{11c}$$

Here, $K_2 = 10$ rad is the gain coefficient, $\tilde{\alpha}(V)$ is the nominal angle of attack, and F is the shear/downdraft factor [Eq. (8c)].

For the ascending branch, the ascent guidance is implemented in the feedback control form

$$\alpha - \tilde{\alpha}(V) = K_3(V - AV_0) + K_4(\dot{V} - 0) \tag{12a}$$

$$\alpha \le \alpha_*, \qquad -\dot{\alpha}_* \le \dot{\alpha} \le \dot{\alpha}_* \tag{12b}$$

Here,

$$K_3 = 0.003 \text{ rad s/ft}, \qquad K_4 = 0.01 \text{ rad s}^2\text{/ft} \tag{12c}$$

are the gain coefficients and $\tilde{\alpha}(V)$ is the nominal angle of attack.

Switch Criteria

In the acceleration guidance scheme, one must define 1) a switch criterion governing the transition from descent guidance to recovery guidance and 2) a switch criterion governing the transition from recovery guidance to ascent guidance. For flight efficiency, one must also introduce 3) a switch criterion governing the transition back from recovery guidance to descent guidance. There is also a bypass criterion: The descent guidance is bypassed if $h_0 \le 0.2h_*$, where $h_* = 1000$ ft.

Criterion 1

The transition from descent guidance to recovery guidance must be performed when a certain target altitude has been reached. Let h_0 denote the initial altitude; let h_T denote the target altitude of the acceleration guidance trajectory; and let $h_{\min}$ denote the minimum altitude of the optimal trajectory. With this understanding, the target altitude can be defined by the relation

$$h_0 - h_T = 0.4(h_0 - h_{\min}), \qquad 0.2h_* \le h_T \le 0.9\,h_0 \tag{13a}$$

which implies that

$$h_T = 0.6h_0 + 0.4h_{\min}, \qquad 0.2h_* \le h_T \le 0.9h_0 \tag{13b}$$

and has the following interpretation: the altitude drop associated with the target altitude of the acceleration guidance trajectory is 40% of the altitude drop associated with the minimum altitude of the optimal trajectory. The lower bound in relations (13) is set in order to avoid ground contact due to excessive undershooting of the target altitude; the upper bound in relations (13) is set in order to avoid early transition from descent guidance to recovery guidance.

We note that the minimum altitude of the optimal trajectory is approximately given by (see Ref. 2 for details)

$$h_{\min} = 0.4h_0 + (0.84 - 4.25\,\dot{W}_x/g)h_*, \qquad 0 \le h_{\min} \le h_0 \tag{14}$$

where $\dot{W}_x$ denotes the average horizontal wind acceleration. As a consequence, the target altitude of the acceleration guidance trajectory becomes

$$h_T = 0.76h_0 + (0.336 - 1.70\,\dot{W}_x/g)h_*, \quad 0.2h_* \le h_T \le 0.9h_0 \tag{15}$$

In actual usage, Eq. (15) must be employed by interpreting $\dot{W}_x$ to be the instantaneous (filtered) horizontal wind acceleration, rather than the average horizontal wind acceleration.

Criterion 2

The transition from recovery guidance to ascent guidance must be performed when the shear region is past and the velocity starts to increase; that is, it must be performed when the instantaneous acceleration $\dot{V}$ switches from negative to positive. An added condition is that the rate of climb be higher than a threshold value, for example, $\dot{h} \ge BV_0$; here, B is a constant to be specified in advance (for instance, $B = 0.05$).

Criterion 3

The transition from recovery guidance back to descent guidance must be performed if, during the recovery phase, the flight altitude has become much higher than the minimum altitude of the optimal trajectory. Therefore, this transition is governed by the simultaneous inequalities

$$h \ge h_{\min} + 0.2h_*, \qquad h \ge 0.3h_* \tag{16}$$

The lower bounds in inequalities (16) mean that this switch is allowed only when sufficient altitude margin exists with respect to the ground.

VI. Gamma Guidance

Here we present a gamma guidance scheme, based on the absolute path inclination, whose objective is to approximate the behavior of the optimal trajectory in a windshear. Just as in the acceleration guidance scheme, the gamma guidance scheme relies on local information on the horizontal wind acceleration, the downdraft, and the state of the aircraft.

The gamma guidance scheme is a modification of the guidance scheme bearing the same name, already developed for takeoff trajectories.[9] The modification accounts for the four facts and ideas of Sec. V.

Guidance Law

The guidance law includes three parts, one pertaining to branch 1 (descent guidance), one pertaining to branch 2 (recovery guidance), and one pertaining to branch 3 (ascent guidance).

For the descending branch, the key property is that the absolute path inclination is negative; its average magnitude increases as the windshear intensity increases; therefore, it increases as the altitude drop associated with the optimal trajectory increases. We surmise that the descent guidance law has the form

$$\gamma_e = \tilde{\gamma}_e \tag{17a}$$

$$\tilde{\gamma}_e = -(h_0 - h_{\min})/Eh_*, \qquad \tilde{\gamma}_e \le 0 \tag{17b}$$

where h_0 is the initial altitude, $h_{\min}$ is the minimum altitude of the optimal trajectory, $h_* = 1000$ ft, and E is a dimensionless constant whose typical value is $E = 2.2$. If Eq. (14) is employed, Eq. (17b) becomes

$$\tilde{\gamma}_e = -(0.6\,h_0/h_* + 4.25\dot{W}_x/g - 0.84)/E, \qquad \tilde{\gamma}_e \le 0 \tag{17c}$$

where $\dot{W}_x$ is the instantaneous horizontal wind acceleration.

For the nearly horizontal branch, the key property is that the absolute path inclination is nearly zero. For strong-to-severe windshears, this property also characterizes the gamma guidance scheme for takeoff trajectories.[9] It follows that the recovery guidance law has the form

$$\gamma_e = \tilde{\gamma}_e \tag{18a}$$

$$\tilde{\gamma}_e = \gamma_{e\tau}[1 - 4(\dot{W}_x/g - W_h/V)], \qquad 0 \le \tilde{\gamma}_e \le \gamma_{e\tau} \tag{18b}$$

Here, $\gamma_{e\tau}$ is the absolute path inclination corresponding to quasisteady steepest climb.

For the ascending branch, the key property is that the instantaneous path inclination tends to a positive value. Here, the path inclination for steepest climb in quasisteady flight is employed. Hence, the ascent guidance law has the form

$$\gamma_e = \tilde{\gamma}_e \tag{19a}$$

$$\tilde{\gamma}_e = \gamma_{e\tau} \tag{19b}$$

Feedback Control

The guidance laws [Eqs. (17–19)] can be implemented via a single feedback control form:

$$\alpha - \tilde{\alpha}(V) = -K(\gamma_e - \tilde{\gamma}_e) \tag{20a}$$

$$\alpha \le \alpha_*, \qquad -\dot{\alpha}_* \le \dot{\alpha} \le \dot{\alpha}_* \tag{20b}$$

Here, $K = 1$ is the gain coefficient; $\tilde{\alpha}(V)$ is the nominal angle of attack; and $\tilde{\gamma}_e$ is nominal absolute path inclination, which is given by Eq. (17c) for the descent guidance, Eq. (18b) for the recovery guidance, and Eq. (19b) for the ascent guidance.

Switch Criteria

In the gamma guidance scheme, one must define a switch criterion governing the transition from descent guidance to recovery guidance and a switch criterion governing the transition back from recovery to descent guidance. There is also a bypass criterion: The descent guidance is bypassed if $h_0 \le 0.2h_*$.

For the gamma guidance scheme, the switch criteria are identical to criteria 1 and 3 of the acceleration guidance scheme [see relations (13–16)]. Although the second switch criterion for the acceleration guidance scheme is necessary for good performance of that scheme, it is not needed in the gamma guidance scheme: The transition from the recovery guidance law [Eq. (18)] to the ascent guidance law [Eq. (19)] occurs automatically at windshear termination; hence, the transition from the recovery feedback law to the ascent feedback law occurs automatically at windshear termination.

VII. Results on Acceleration and Gamma Guidance

Using the data of Sec. II, numerical results were obtained for the acceleration guidance scheme and the gamma guidance scheme. Several combinations of initial altitudes and windshear intensities were considered [see Eqs. (2) and (3)]. From the numerical solutions, upon comparing the acceleration guidance trajectory (AGT), the gamma guidance trajectory (GGT), and the optimal trajectory (OT), certain general conclusions are apparent:

1) The function $h(t)$ of AGT and the function $h(t)$ of GGT are close to the function $h(t)$ of OT. The trajectories of both the AGT and the GGT include a descending branch, followed by a nearly horizontal branch, followed by an ascending branch, just as the OT.

2) For both the AGT and the GGT, the peak altitude drop depends on the initial altitude and the windshear intensity, increasing as the initial altitude and the windshear intensity increase. This property is consistent with the analogous property of the OT.

3) The function $V(t)$ of the AGT and the function $V(t)$ of the GGT are close to the function $V(t)$ of the OT. For both the AGT and the GGT, the point of minimum velocity is achieved near the end of the shear ($t = 22$ s). In the range of initial altitudes and windshear intensities considered, the minimum velocity is nearly independent of h_0 and λ. Also, the minimum velocity is near to (but is not equal to, because of dynamical effects and windshear effects) the stick-shaker velocity in quasisteady level flight, $V_* = 179.3$ fps.

4) For strong-to-severe windshears, the functions $\alpha(t)$ of the AGT and the GGT resemble the function $\alpha(t)$ of the OT: the angle of attack exhibits an initial decrease, followed by a gradual, sustained increase; the peak value of the angle of attack is achieved near the end of the shear. The function $\alpha(t)$ of the OT is smoother than the functions $\alpha(t)$ of the AGT and the GGT. For both the AGT and the GGT, relatively large oscillations of the angle of attack occur near the point of switch from descent guidance to recovery guidance. These oscillations can be reduced by employing more sophisticated forms of feedback control than the simple proportional feedback control considered here.

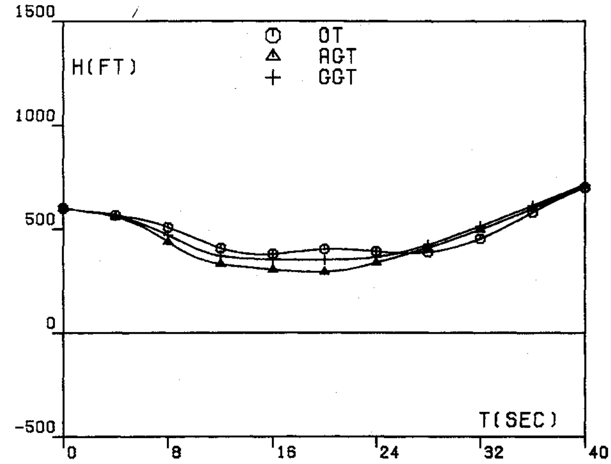

Fig. 2a Trajectory comparison, $h_0 = 600$ ft, $\lambda = 1.2$: altitude h vs time t (OT, AGT, GGT).

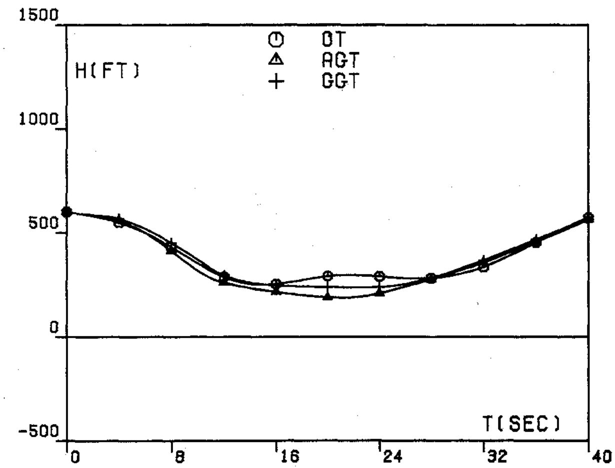

Fig. 2b Trajectory comparison, $h_0 = 600$ ft, $\lambda = 1.4$: altitude h vs time t (OT, AGT, GGT).

5) For strong-to-severe windshears, the functions $\theta(t)$ of the AGT and the GGT resemble the function $\theta(t)$ of the OT. For both the AGT and the GGT, the pitch attitude angle maintains relatively small values in the initial branch of the trajectory; then, it increases gradually in the central branch; larger values of the pitch attitude angle are achieved toward the end of the shear. For both the AGT and the GGT, relatively large pitch attitude oscillations occur near the point of switch from descent guidance to recovery guidance, induced by the corresponding angle-of-attack oscillations. These oscillations can be reduced by employing more sophisticated forms of feedback control than the simple proportional feedback control considered here.

6) Even though both the AGT and the GGT are close to the OT, it is felt that the acceleration guidance trajectory is to be preferred, because of the following reasons: The AGT is relatively insensitive to time delays in windshear detection if the prewindshear guidance $V = $ const is employed; this is not the case with the GGT, due to the switch from one guidance law to another; the descent guidance of the AGT is easier to implement than the descent guidance of the GGT.

For a particular case, namely, $h_0 = 600$ ft, comparative numerical results concerning the functions $h(t)$ of the OT, the AGT, and the GGT are presented in Fig. 2. This figure includes two parts: Fig. 2a refers to $\lambda = 1.2$, and Fig. 2b refers to $\lambda = 1.4$; for more detailed results, see Ref. 2.

VIII. Theta Guidance

In this section, we present a theta guidance scheme (modified constant pitch guidance) designed to achieve improvements over the constant pitch guidance scheme of Refs. 3–5.

Constant Pitch Guidance

The constant pitch guidance is based on the target pitch

$$\theta=\theta_* \tag{21}$$

where $\theta_*=17.0$ deg for the Boeing B-727 aircraft. Because $\theta=\alpha+\gamma$, the angle-of-attack law is given by

$$\alpha=\theta_*-\gamma \tag{22a}$$

$$\alpha\leq\alpha_*, \qquad -\dot{\alpha}_*\leq\dot{\alpha}\leq\dot{\alpha}_* \tag{22b}$$

Theta Guidance

The theta guidance (modified constant pitch guidance) attempts to incorporate some of the properties of the optimal trajectories into the constant pitch guidance. As noted in Sec. IV, for strong-to-severe windshears, the pitch attitude angle maintains relatively small values in the neighborhood of the initial point; then, it increases gradually in the nearly horizontal branch of the trajectory; larger values of the pitch attitude angle are achieved near the end of the shear. This suggests the introduction of two different target pitches as follows:

Branch 1:

$$\theta=\theta_1 \tag{23a}$$

Branch 2:

$$\theta=\theta_2 \tag{23b}$$

The lower target pitch can be chosen to be $\theta_1=0$ or $\theta_1=\theta_0$; the higher target pitch is chosen to be $\theta_2=\theta_*$. Because $\theta=\alpha+\gamma$, the angle-of-attack law is given as follows:

Branch 1:

$$\alpha=\theta_1-\gamma \tag{24a}$$

$$\alpha\leq\alpha_*, \qquad -\dot{\alpha}_*\leq\dot{\alpha}\leq\dot{\alpha}_* \tag{24b}$$

Branch 2:

$$\alpha=\theta_2-\gamma \tag{25a}$$

$$\alpha\leq\alpha_*, \qquad -\dot{\alpha}_*\leq\dot{\alpha}\leq\dot{\alpha}_* \tag{25b}$$

Switch Criteria

In the theta guidance, one must define a switch criterion governing the transition from descent guidance to recovery guidance. For flight efficiency, one must also introduce a switch criterion governing the transition back from recovery guidance to descent guidance. The switch criterion governing the transition from recovery guidance to ascent guidance is not needed. There is also a bypass criterion: The descent guidance is bypassed if $h_0\leq 0.2h_*$.

For the theta guidance scheme, the switch criteria are identical to switch criteria 1 and 3 of the acceleration guidance scheme [see relations (13–16)].

IX. Results on Theta Guidance

Using the data of Sec. II, numerical results were obtained for both the constant pitch guidance and the theta guidance (modified constant pitch guidance). The constant pitch guidance was implemented via relations (22), with $\theta_*=17.0$ deg. The theta guidance was implemented via relations (24) and (25), with $\theta_1=\theta_0$ and $\theta_2=\theta_*=17.0$ deg.

Several combinations of initial altitudes and windshear intensities were considered [see Eqs. (2) and (3)]. From the numerical results, upon comparing the constant pitch trajectory (CPT) and the modified constant pitch trajectory (MCPT), certain general conclusions become apparent:

1) The function $h(t)$ of the MCPT includes a descending branch, followed by a nearly horizontal branch, followed by an ascending branch, just as the OT. On the other hand, the nearly horizontal branch has almost disappeared from the CPT.

2) The minimum altitude of the MCPT is achieved earlier than the minimum altitude of the CPT. Also, the minimum altitude of the MCPT is higher than the minimum altitude of the CPT.

3) The minimum velocity of the MCPT is greater than the minimum velocity of the CPT. More importantly, as in the OT, the minimum velocity of the MCPT is achieved near the end of the shear.

4) For strong-to-severe windshears, the function $\alpha(t)$ of the MCPT has an initial decrease, followed by a gradual, sustained increase; the peak value of the angle of attack is achieved near the end of the shear. On the other hand, the function $\alpha(t)$ of the CPT does not have an initial decrease. The stick-shaker angle of attack $\alpha=\alpha_*$ is maintained for a longer time interval in the CPT than in the MCPT.

5) For strong-to-severe windshears, the function $\theta(t)$ of the MCPT maintains $\theta=\theta_0$ before the transition, followed by $\theta=\theta_*$ after the transition. Although the target pitch θ_* can be maintained in the MCPT, it cannot be maintained in the CPT due to angle-of-attack saturation.

For a particular case, namely, $h_0=600$ ft, comparative results concerning the functions $h(t)$ of CPT and the MCPT are presented in Fig. 3. This figure includes two parts: Fig. 3a refers to $\lambda=1.2$, and Fig. 3b refers to $\lambda=1.4$; for more detailed results, see Ref. 2.

X. Survival Capability

In this section, we analyze the survival capability of an aircraft in a severe windshear. Indicative of this survival capability is the windshear/downdraft combination that results in the minimum altitude being equal to the ground altitude.

To analyze this important problem, we recall the one-parameter family of windshear models [Eqs. (13–14) of Ref.

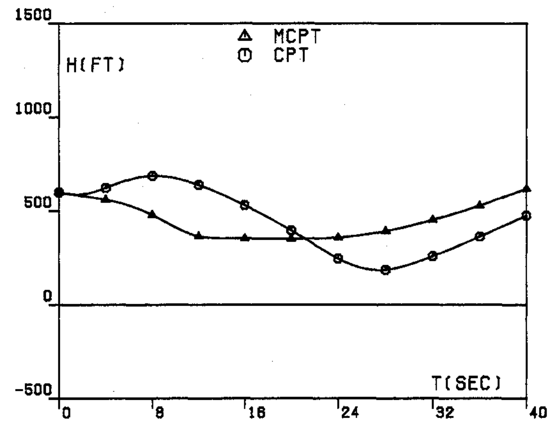

Fig. 3a Trajectory comparison, $h_0=600$ ft, $\lambda=1.2$: altitude h vs time t (CPT, MCPT).

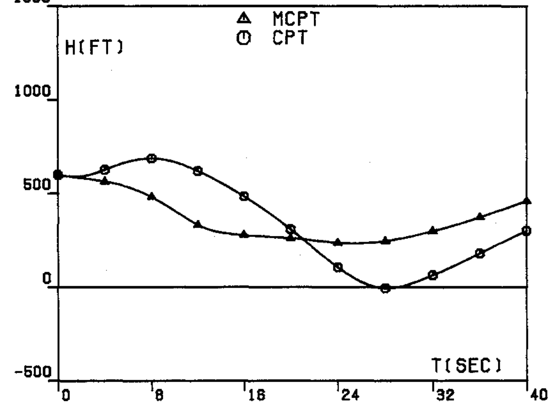

Fig. 3b Trajectory comparison, $h_0=600$ ft, $\lambda=1.4$: altitude h vs time t (CPT, MCPT).

Table 1 Survival capability: critical wind velocity difference ΔW_{xc} (fps)

h_0, ft	OT	AGT	GGT	CPT	MCPT
200	157.8	137.8	134.8	125.2	125.2
600	187.1	179.1	184.2	139.4	172.0
1000	210.7	199.3	197.5	153.6	181.7

Table 2 Survival capability: windshear efficiency ratio (WER)

h_0, ft	OT	AGT	GGT	CPT	MCPT
200	1.000	0.873	0.854	0.793	0.793
600	1.000	0.957	0.985	0.745	0.919
1000	1.000	0.946	0.937	0.729	0.862

1], in which the parameter λ characterized the intensity of the windshear/downdraft combination. By increasing the value of λ, more intense windshear/downdraft combinations are generated until a critical value λ_c is found (hence, a critical value ΔW_{xc} is found), such that $h_{\min}=0$ for a given guidance scheme.

More precisely, we consider the OT, the AGT, the GGT, the CPT, and the MCPT. The results are shown in Table 1, which supplies the critical wind velocity difference ΔW_{xc}, and in Table 2, which supplies the windshear efficiency ratio WER, defined to be

$$\mathrm{WER} = (\lambda_c)_{\mathrm{PT}}/(\lambda_c)_{\mathrm{OT}} = (\Delta W_{xc})_{\mathrm{PT}}/(\Delta W_{xc})_{\mathrm{OT}} \tag{26}$$

Here, the subscript PT denotes a particular trajectory and the subscript OT denotes the optimal trajectory. From Tables 1 and 2, the following conclusions can be inferred:

1) The survival capability of the AGT and the survival capability of the GGT are close to the survival capability of the OT.

2) The survival capability of the MCPT is better than the survival capability of the CPT and is closer to the survival capability of the OT.

3) In terms of survival capability, if one defines the efficiency ratio of the OT to be 100%, then that of the AGT is 87–96%; that of the GGT is 85–98%; that of CPT is 73–79%; and that of the MCPT is 79–92%.

XI. Conclusions

The guidance of abort landing trajectories in the presence of windshear has been studied under the assumption that the maximum power setting is employed so that the only control is the angle of attack. Two guidance schemes have been developed: acceleration guidance (based on relative acceleration signals) and gamma guidance (based on absolute path inclination signals).

Both guidance schemes produce trajectories mirroring the behavior of the optimal trajectories. These trajectories include a descending flight branch, a nearly horizontal flight branch, and an ascending flight branch. The transition from descent guidance to recovery guidance is governed by a switch signal, depending on the initial altitude and the windshear intensity.

A theta guidance scheme (modified constant pitch guidance) has also been developed that employs the idea of two target pitches: a lower target pitch, useful for descent guidance, and a higher target pitch, useful for recovery guidance/ascent guidance.

Computer experimentation with the Boeing B-727 aircraft shows that the acceleration guidance, the gamma guidance, and the theta guidance are superior to the constant pitch guidance in terms of altitude loss and survival capability in severe windshears, albeit at the expense of additional instrumentation.

If one compares optimal trajectories with constant pitch trajectories, it appears that the gains in survival capability are of the order of 15% for the takeoff problem and 25% for the abort landing problem. This suggests that the advantages to be realized by the development of windshear guidance systems are potentially higher for the abort landing problem than for the takeoff problem.

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